29 January 1962 L-2044

Dr. I. J. Good
Admiralty Research Laboratory
Teddington, Middlesex
ENGLAND

Dear Dr. Good:

I have just come across references, in your comments on C. A. B. Smith's article, "Consistency in Statistical Inference and Decision," to two recent papers by you which I am extremely anxious to obtain. One is in the Proceedings of the International Congress for Logic, Methodology and Philosophy of Science, 1960; the other, in the Fourth London Symposium on Information Theory. Since neither of these seems to be available yet in this country, I would appreciate very much receiving reprints of these papers, if any are available. In fact, since I am planning to present a Ph.D. thesis on March 1st (to Harvard) on a subject to which I suspect these papers have the highest pertinence, I would be happy to pay the cost of having them sent airmail.

I am enclosing a copy of a recent paper of mine which should make clear the reasons for my interest in your work. Since writing it -- unfortunately not before -- I came across your 1952 article, "Rational Decisions," which led me to track down everything I could find of your earlier and subsequent writings. What leaped out at me in "Rational Decisions" was your clear discussion (p. 114) of the same decision rule analyzed in the paper I had just published, a rule which I had seen previously described only by Hurwicz and by Hodges and Lehman (in papers cited in my enclosed article). The "wooliness," "vagueness" or "lack of definition" of subjective probabilities which you suggest as the motivation for this rule seems closely related to the "ambiguity" which I discuss; and your notion (like that of Koopman) of subjective probabilities as being bounded by inequalities expresses the notion of "ambiguity" quite precisely. Although your discussion is almost always in terms of relations between beliefs, whereas my argument, following that of Savage and Ramsey, focuses upon relations between actions, I think the relationship between the two approaches is clear. Those situations in which I assert (in the paper) that reasonable people often act in violation of the Savage axioms are undoubtedly those in which their probability judgments are in the form of inequalities, with significant intervals between what Koopman calls their "upper and lower probabilities."

The enclosed paper really represents my thinking as of 1957-1958. Since publishing it at last, this fall, I have been developing the subject further as a doctoral thesis. Whereas my earlier approach had been entirely from the point of view of decision theory, I have lately been particularly stimulated by the work on intuitive probability by Keynes, Koopman, and yourself.

My article does confront directly a problem which you have not discussed explicitly, one on which I would be most interested to have your comments. The "Type II minimaxing" you describe in several contexts does have the implication that it would lead to violations of the Savage postulates, specifically his Postulate 2, the Sure-Thing Principle. The question I consider is whether Type II minimaxing can, in view of this consequence, still be regarded as "reasonable." I argue that it can. I suspect, from your comments on "taking inequalities seriously" that you would agree. Still, my position remains a lonely one in the published literature; I am not aware of any other published statement even suggesting that it might be acceptable in any circumstances to violate Savage's Sure-Thing Principle, let alone identifying any specific circumstances in which this might be reasonable. Your own recent comments on the importance of inequalities and the reasonableness of the behavior described by Smith encourages me to believe that you are prepared to part company with the Sure-Thing Principle in situations where intuitive probabilities are "non-appraisable."

The issue seems to me to be this. Suppose that, as you put it, the initial probabilities are "'known' only to lie in wide intervals"; the question arises, How may one reasonably act on the basis of such vague estimates? Certain decision rules with respect to these interval estimates of probabilities would not lead to violation of the Savage axioms: for example, the rule of maximizing expected utility with respect to the probabilities at the mid-point of each interval. Other decision rules, such as the one you label "Type II minimaxing," would lead to violations of the Savage axioms. Savage, in effect, argues that the latter rules are to be rejected as leading to unreasonable behavior. I do not agree; nor, I take it, do you. Smith's position on this is not quite clear to me; if I understand his implied decision rule correctly (and I am not sure that I do), he suggests that one pick a probability estimate randomly from the interval of reasonable estimates. This would lead to violations of Savage's Postulate 1, but not, I suspect, to closely related sets of stochastic axioms such as those proposed by Luce.

It happens that I once had a brief opportunity to present my views to you. You probably don't remember the occasion, but it was an historic day for me, since I had just an hour earlier confronted Savage with my examples and won from him (to the surprise of both of us) the admission that he would wish in these circumstances to violate his own postulates. (He recently commented that he still feels the intuitive "pull" of the example, though he reserves judgment as to the implications of this for the axioms.) As I remember, you were just preparing to leave Chicago at that moment, and Savage brought me over to the Faculty Club to test your reactions to the same questions. To the best of my recollection (this was, I believe, about February 1958), your immediate responses did not violate the axioms, though this scarcely dampened my spirits on that particular day. I would be delighted to find, of course, that you had changed your mind.

Very truly yours,

Daniel Ellsberg
Economics Department

DE/ss
Enclosure

P-2173, <u>Risk, Ambiguity, and the Savage Axioms</u>, dtd. 8/25/61

bcc: Reports
B. Haydon